Validation of FE Models Using Experimental Data and Monte Carlo Simulation

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Abstract

In the past few years, there has been growing belief that High Performance Computing, together with Computer-Aided Engineering, can help eliminate the expensive testing campaigns that most manufacturing companies are using to help develop their products. However, the key technological component that can make the reduction of physical prototyping possible, namely the validation of numerical models, has not, surprisingly, received due attention. Model validation is instrumental towards making models reliable, which is, clearly, fundamental towards truly predictive digital prototyping. If one does not know how to *quantify* the amount of confidence behind a numerical model, eliminating physical prototyping is risky and, at best, disputable. The paper discusses a novel approach to model validation based on stochastic techniques. It is shown how simple concepts from statistics can be of great help towards establishing the pedigree of an FE model.

1 Introduction and Background

It is quite evident that CAE has reached a phase of saturation and stagnation. Many companies, after substantial investements in HPC hardware and software, are forced to admit that CAE has not lived up to expectations. The market is overpopulated with FE codes and tools for design and topology optimization, but the issue of model validity, a highly embarassing one it seems, is silently set aside. Under similar conditions, how can one seriously propose to eliminate a physical test with a digital simulation if he cannot say that his model has, say, a 95% level of confidence, or credibility, attached to it. Evidently, being able to formulate a similar statement has many profound implications, as the paper shall illustrate.

The existence of scatter, as it appears, has now been widely accepted and is no longer under discussion. The uncertainty behind material properties, loads, boundary conditions and manufacturing and assembly tolerances, is the main reason why two supposedly identical experiments will yield different results. Two cars coming from the same production line, on the same day, will exhibit different crashworthiness when tested even in the same lab and by the same people. This fact is well known and yet, surprisingly, engineers are involved in building very detailed models with the intention of capturing the results of one particular test. It is difficult to understand what exactly is "detailed" in these models, given the fact that the real world is neither detailed, nor accurate, nor precise. Now, since tests are, for very good physical reasons, non-repeatable, this "accurate chase" is lost from the very beginning. A model can be made to reproduce a certain experiment with arbitrarily small error (at least in theory), but when the experiment is repeated, the results change and therefore, the model is no longer valid. So what is, then, model validity? Is the fortunate one-to-one correspondance (not correlation!) of results sufficient to say that a model is valid? Clearly, the answer is yes if the particular phenomenon in question is 100% repeatable. In all other cases, this only provides a weak and necessary condition. When dealing with non-repeatable, or stochastic phenomena, it is necessary to prove a certain number of times, that beyond "reasonable doubt", the numerical model mimicks the experiment. And it is also necessary, in order to add a fundament of mathematical rigour to the whole matter, to be able to quantify the "degree of belief" in the model's performance through some computable quantity. In summary, a numerical model must also exhibit the same scattter as one observes in repeated experiments. Therefore, the concept of model must be broadened in order to embrace also this other important facet of physics, namely uncertainty. Uncertainty, however, has been axiomatized out of modern CAE. Hardware and software companies, in attempts to secure future businesses, have pushed the industry into thinking that the pursuit of detail leads to the understanding of the whole (this is known as reductionism, based, in turn on determinism). Quick growth of computer speed and memory has left room for engineers to merely increase the size of their models, leaving behind many aspects of physics. Uncertainty is one them. Today, CAE concentrates on *repeatable details*. In the future, thanks to stochastic techniques, emphasis shall be placed upon *repeatable patterns*. The management of uncertainty is the key to the understanding of complexity, its patterns, and the order and knowledge it hides.

Today's techniques of model-to-test comparison, are not sufficiently rigorous and sound enough to even vaguely hint anything on the validity of these models, not to mention help reduce physical prototyping. But what is the direction to follow in order to make test reduction a real possibility? Traditionally, in any large manufacturing company, the test department and CAE department form, due to some unknown reason, separate entities, often physically apart. They very often disagree on how a test should be performed, and both distrust and disbelieve the other party's results. In order to reduce prototyping, and therefore to ensure CAE quantifiable credibility, it is first necessary to bring testing and CAE to a common conceptual and technological platform. Clearly, the number of prototypes may be drastically reduced (probably not eliminated, at least for some time to come) if the physics behind the phenomenon in question is *fully* understood and appreciated. This requires close interaction, not separation. One very good reason, however, for the separation between experimentation and CAE is the fact that until very recently there was no established methodology, or technology, to enable rigorous model validation.

The first known large-scale EU effort to investigate model validation via comparison with experimental data has been the ESPRIT HPCN-SCAT Project, conceived and lead by the author in the period 1997-99. SCAT, Stochastic Correlation of Analysis and Test, had the objective of proposing an innovative methodology for rigorous correlation between experiments and Monte Carlo Simulation (MCS). The technology developed in HPCN-SCAT is currently being implemented in EASis stochastic analysis tool ST-ORM. Details are provided in the following section.

2 What is Model Validation

Monte Carlo Simulation is an extremely powerful methodolgy (not just an algorithm!). In fact, even Nature acts as if it were running a Monte Carlo Simulation, in that always populations are created, never isolated entities or organisms. Now, let us get back to our repeated tests. Let us suppose that we are performing a number of vibration tests on some sort of structure that is manufactured in series. Let us also limit, without loss of generality, the number of vibration modes of interest to just three. Due to the uncertainties (tolerances), each test shall yield three values of frequencies that will, with high probability, differ from those measured in the preceeding experiments (realisations). If we now plot the result of each experiment, as a point with three coordinates (i.e. the modal frequencies) we will find that the points form a three-dimensional cloud. If we repeat the experiment a number of times that allows the cloud to "settle down" and stabilize, in terms of position, shape, orientation, etc., we may state that this cloud is *the model* of our expriment (really of a series of experiments). It is the whole cloud now that carries the information on the physics of the phenomenon, not a single point. In reality, these clouds, called *response clouds*, or *meta-models*, can be extremely complex and, of course, can occupy a large number of dimensions. Statistical techniques, such as the Principal Component Analysis (PCA) can help to determine which directions in these multi-dimensional constellations are of particular importance and which are not. PCA helps to determine something similar to the principal axes of inertia of a solid, and is therefore key towards the understanding of the nature of response clouds.

Let us now turn our attention to CAE. Suppose that we build a Finite Element Model of our structure. Using Monte Carlo Simulation on a computer (the tesing campaign just described is experimental Monte Carlo) we can also generate response clouds. In very rare occasions, the measured and computed response clouds coincide. Sometimes, their position in the multi-dimensional response space is almost equivalent, but their orientations are not. In other cases, the positions are distant, while the shapes look very much alike. The question of model validation is now clear. The two clouds must be made to overlap. At least as far as position of the center of gravity, the shape and orientation are cocerned. We could consider features of higher order, like kurtosis or skewness (reflecting cloud density or aspect ratios in certain directions). However, experience suggests, that satisfying only the first three features (position, shape and orientation) is already beyond the reach of modern CAE. In fact, our modelling and simulation capability has not yet enabled a "computed cloud" to match a measured cloud at the first attempt. EASi's ST-ORM tool (STochastic Optimisation and Robustness Management) provides the necessary tools to actually move, shape and orient response clouds in their multi-dimensional spaces. However, as the CAE communinity may rightly say, why always trust a test (or a series of tests for that matter) 100%? Clearly, it is necessary to establish a meeting point between the two disciplines, and with high probability the meeting point lies somewhere in the middle. In any case, once we have confidence that the two candidate clouds are equivalent, we must establish some quantities that help to express the level of equivalence. Without getting into further details (see [2] and [3]) it is possible to express the measure of cloud equivalence (or distance), even between clouds having a different numbe of points, as follows:

1. The Mahalanobis distance. This metric, based on the pooled covariance matrix, is a measure of the distance between the centers of gravity of the two clouds.

2. The Principal Components. These are eigenvectors of the covariance matrix and reflect the orientation of each cloud.

3. The spectral properties (eigenvalues) of the covariance matrices. The eigenvalues of the covariance matrices express the aspect ratios of the two clouds.

It is clear, therefore, that the validation of the two clouds (it is not important to say now which one with respect to which!) requires covariance matrices, and these, in turn, require repeated realizations, be it experiment or simulation. Quite expensive, in case we're talking of testing! Isn't test number reduction what we're all after? Here, we're saying that model validation actually requires an increase in testing! Isn't this a bit of a logical inconsistency? In actual fact, today's CAE is in logical fault and ill-equipped to help reduce physical prototyping. If, and only if, CAE will be able to deliver, at first attempt, computed clouds that will overlap in a statistically significant manner, then we can talk *confidently* of test number reduction. Until that happens, we must first invest into actually increasing the volume of testing and help validate, *once and for all* our numerical models. CAE has lived for a very long time outside of physics. The excessive focus on computers and numerical aspects has beatified overly complex software tools which have progressively lost touch with reality. It is time to pay our debt with physics. The dreamed of digital prototyping nirvana lies on a plateau. We are at the moment at the foot of a mountain separating us from this plateau. Companies must understand that in order to make the most of CAE's fantastic potential, we must first of all pay due respect to physics, and understand well what is it exactly that we're talking of reducing. If we want, for example, to reduce drastically crashworthiness testing, we must be first of all able to understand the physics behind car crash, so that the elimination of a test is done with a low, and known, level of risk. If that means having to crash a hundred cars first, then we must be prepared to pay the price. But we only need to do it once. Test elimination cannot come for free. Nature offers no free lunch!

In many cases, it has been observed that computed response clouds change shape when one changes, for example, mesh size, time-step-size, algorithm type, computer, solver, etc. Clearly, one important step towards having a valid response cloud, is to make it first immune to solver-specific parameters before actually attempting correlation with measured response clouds. Monte Carlo Simulation can help to identify the regions of these "non-physical" quantities where their changes will not be reflected in the response cloud's statistical properties. Once these regions have been identified, we can proceed to concentrate on physics. In practice, we need first to separate physics from "numerics" in order to talk of a sound and reliable model. Today's commercial CAE tools are very complex. They offer many algorithms and procedures, all based on certain simplifications and assumptions, that produce uncertain effects on the overall solution. On very rare occasions, is the impact of these simplifications (such as those induced by the DOE or by the response surface method) assessed and quantified, and it normally blends, in an uncertain manner, with the physics the model is trying to convey. Monte Carlo Simulaton, on the other hand, enables to first "filter-out" the impact of a particular modelling technique and to provide a healthy numerical model.

Statistical equivalence of two clouds, even if taken to high order moments, still only provides a necessary condition for model validity. We might, in fact, be missing a certain layer of physics in our model (for example the dependency on temperature or humidity) and still be able to "force" the computed cloud to match well the measured one. Computers allow a lot of freedom in this sense. A sufficient condition is always more expensive. We must, however, satisfy the necessary one first.

3 Conclusions

The paper has shown that in order to be able to validate a numerical model it is first necessary to redefine the concept of model. A model, in fact, is a cloud of responses, either measured or computed. It is only a model of this type that can be validated. Secondly, it is necessary to possess two such models in order to speak of statistical equivalence. In practice, this means that the sought-after reduction of number of prototype testing requires an initial investment in terms of repeating tests. This investment needs to be made only once and will enable CAE to deploy its full potential. In practice, CAE and testing must be first brought to a common platform before one can replace the other. The procedures described herein establish a methodology whereby CAE and physics are linked much more closely than it has been possible in contemporary complex and numerically-oriented software tools. Monte Carlo Simulation establishes such a platform. Its great advantage lies in its simplicity and genericity. Today, with the abundance of low-cost High Performance Computing hardware, there is no further justification to resort to approximate techniques whose impact on simulation cannot be determined and discriminated.

References

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